Application of the Genetic Algorithm for Microwave Imaging of a Layered Dielectric Object Via the Regular Shape Expansion Technique

Ching-Lieh Li, Yu-Yi Cheng

Department of Electrical Engineering, Tamkang University, Tamsui, Taipei Hsien, Taiwan 25137

ABSTRACT: A novel method combining the genetic algorithm (GA) and regular shape expansion technique is reported for electromagnetic imaging of a multilayer dielectric object of arbitrary shape. By measuring the scattered field, the shape, location, size, and permittivity of each layer of the object are retrieved quite successfully. The forward problem is solved based on the equivalent source current and the method of moments (MoM), while the inverse problem is reformulated as an optimization problem. The optimization problem is solved by the proposed method. Numerical simulation shows that good image reconstruction can be obtained for various multilayer dielectric objects as long as the noise level is ≤ -20 dB. © 1999 John Wiley & Sons, Inc. Int J Imaging Syst Technol, 10, 347–354, 1999

Key words: microwave image; inverse scattering; genetic algorithm; regular shape expansion technique; multi-layer dielectric object

I. INTRODUCTION

A novel method to retrieve the shape, location, size, and the internal property of an object embedded in a homogeneous space or buried underground via electromagnetic scattering has gained increasing interest in many areas such as nondestructive evaluation, geophysics, biomedical application, material engineering, and environmental investigations. We have previously reported a method combining the genetic algorithm (GA) and shape mutation scheme for inverse scattering of a homogeneous dielectric cylinder (Li and Cheng, 1998). In this paper, we propose a new method combining the GA with the regular shape expansion technique to reconstruct the microwave image of various layered and/or piecewise homogeneous dielectric objects. The inverse problem is easier when the object is homogeneous and the permittivity is known and when some priori topological information is available. When a layered object is considered and the permittivity is assumed unknown, the shape identification problem is difficult, especially when no priori topological information is available, because the relationship between the scattered field and the shape and/or permittivity of the layered object is strongly nonlinear. We focus on the identification of a two-dimensional (2-D) penetrable layered object embedded in a homogeneous space. A TM electromagnetic wave is cast in a test object domain to characterize the permittivity, shape, location, and size of each layer of the object.

Most traditional iterative algorithms for inverse scattering are founded on a functional minimization via some gradient-type scheme, such as the Newton-Kantorovitch method (Roger, 1981; Tobocman, 1989; Chiu and Kiang, 1991), the Levenberg-Marquardt algorithm (Colton and Monk, 1986; Kirsch et al., 1988; Hettlich, 1994), or the conjugate-gradient method (Kleinman and Van Den Berg, 1994). In general, during the search of the global minimum, they tend to get trapped in local minima when the initial guess is far from the exact one. Besides, some prior topological information or regulation methods are used to overcome the illposedness. Performance of these approaches can be further ameliorated against local minima by introducing alternative expansions of the unknown complex permittivity profile (Isernia et al., 1997; Maniatis et al., 1998). When only lesser topological constraints can be enforced, there are methods that use level-set modeling reported (Santosa, 1996; Litman et al., 1997).

In this paper, the GA is applied to reconstruct a layered dielectric object through the regular shape expansion technique. There are several advantages for employing the GA for the inversion purpose. First, the GA is robust in its capability to reach the global minima and is insensitive to fitness function details. Moreover, it is easy to insert a lot of prior information into the solving procedure. In other words, the GA is a global optimization technique based on the evolutionary mechanism of natural selection. It searches through a coded parameter space randomly using three main operations, (i.e., crossover, mutation, and, selection) and tends to yield global extreme solution (Goldberg, 1989; Johnson and Rahmat-Samii, 1997). In accordance with the regular shape expansion technique, the GA is an appropriate method for the inversion of the multilayer dielectric objects investigated. Section 2 presents the theoretical formulation and the inverse scheme. The idea of regular shape expansion is introduced to work with the GA. Thus, the proposed method can be successfully applied to reconstruct the microwave image of objects consisting of multiple layers. In Section 3, various dielectric objects with arbitrary cross-section shape are considered. The numerically experimental results for shape and permittivity reconstruction of each layer are quite good. The effects of random noise on the inversion are also reported. Good inversion is ensured as long as the noise level is ≤ -20 dB.

II. THEORETICAL FORMULATION AND INVERSE SCHEME

Consider a multilayer dielectric object of arbitrary cross-section in free space (Fig. 1) in which the object is assumed infinitive long in

Correspondence to: C.-L. Li
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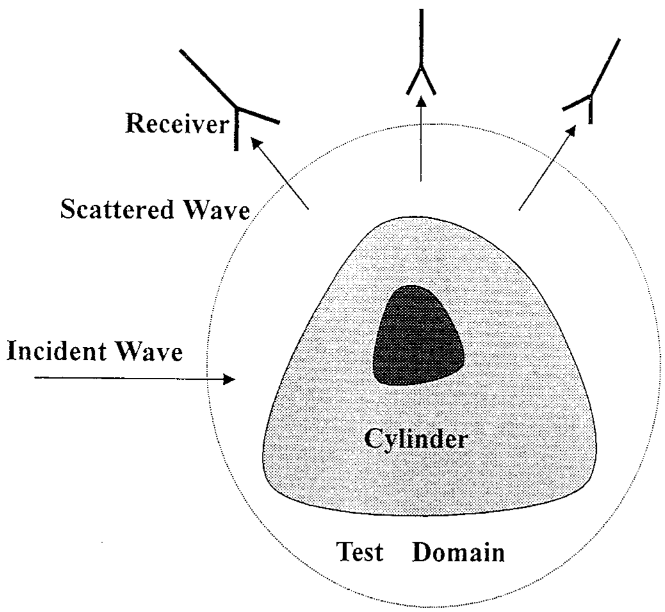

Figure 1. Geometry of a multilayered dielectric object in free space illuminated by a TM plane electromagnetic wave.

z-direction. The object is illuminated by an incident electromagnetic plane wave of TM polarization, i.e., $\vec{E}$ is parallel to the z-axis. The forward problem can be analyzed using the concept of equivalent current $\vec{J}_{eq}$ such that the scattered field outside the cylinder can be expressed by

$$E_z^s(\vec{r}) = \int_s G(\vec{r}, \vec{r}\,')k_0^2(\varepsilon_r - 1)E_Z(\vec{r}\,')ds' \tag{1}$$

where k_0 is the wavenumber in free space and

$$G(\vec{r}, \vec{r}\,') = \frac{-j}{4} H_0^{(2)}(k_0|\vec{r} - \vec{r}\,'|) \tag{2}$$

is the 2-D Green's function for the free space, and $H_0^{(2)}$ is the Hankel function of the second kind with order zero. E_Z is the total internal field inside the object that can be solved by the following EFIE

$$-E_Z^i(\vec{r}) = \int_s G(\vec{r}, \vec{r}\,')k_0^2(\varepsilon_r - 1)E_z(\vec{r}\,')ds' - E_z(\vec{r}) \tag{3}$$

For the forward problem, the method of moments (MoM) is applied to solve Eq. (3) and then the scattered field E_z^s is calculated by Eq. (1) when the geometrical parameters and the dielectric properties of the layered object are given. Conversely, if we obtain sufficient information from the measured scattered field, we should be able to reconstruct the permittivity, shape, location, and size of each layer of the object.

To numerically calculate the scattered field, we first divide the test domain of dielectric object into N sufficient small cells such that the relative permittivity distribution and the electric field can be written as

$$\varepsilon_r(x', y') = \sum_{n=1}^{N} \varepsilon_{rn} P_n(x', y') \tag{4}$$

$$E_Z(x', y') = \sum_{n=1}^{N} E_{zn} P_n(x', y') \tag{5}$$

where P_n, $n = 1 \sim N$ are the pulse functions.

Then, using the MoM with point matching procedure, we can transform the EFIE (3) and Eq. (1) into a system of algebraic equations, respectively, as follows:

$$-[E_z^i] = \{[G_1][\tau_Z] - [I]\}[E_z] \tag{6}$$

and

$$[E_z^s] = [G_2][\tau_Z][E_z] \tag{7}$$

where $[E_z]$ and $[E_z^s]$ represent the N element total field column vector and the M element scattered field column vector, respectively. Here M is the number of the measurement points. The elements of $[G_1]$ and $[G_2]$ can be obtained via tedious mathematical manipulation. $[G_1]$ is an $N \times N$ square matrix and $[G_2]$ is a $M \times N$ matrix. $[\tau_Z]$ is an $N \times N$ diagonal matrix with diagonal element $\tau_{Zn} = \varepsilon_{rn} - 1$.

Equations (6) and (7) are called the forward scattering formulas. If the permittivity of each cell is given, the total field inside the test domain and the outside scattered field can be calculated directly.

For the inverse problem, we have previously reported a method combining the GA with the shape mutation scheme to reconstruct hollow or multicylinder homogeneous objects (Li and Cheng, 1998). Using the shape mutation scheme along with the GA, we have successfully applied the method to retrieve the shape, size, and permittivity of various homogeneous dielectric objects starting from the initial population with a uniform permittivity profile. However, it is recognized that the number of unknowns increases as the division number of test domain increases such that the converging rate would become unacceptable, eventually. In this paper, a new method combining the idea of the GA with the shape expansion scheme is proposed to solve the above problem. In addition, this method is especially suited for multilayer dielectric objects.

It is well known that the GA is an optimization technique that models the natural processes of evolution and selection via genetic recombination. In general, a GA optimizer must be able to perform seven basic tasks:

1. Encode the solution parameters as genes
2. Create a string of genes to form a chromosome
3. Initialize a starting population
4. Evaluate and assign fitness values to individuals in the population
5. Perform reproduction through some selection scheme
6. Perform recombination of genes to produce offspring
7. Perform mutation of genes to produce offspring

A typical flow chart of the GA is given in Figure 2. The searching process stops when some preset criteria are met such as (1) a desired precision is achieved, (2) the fitness value is not improved for a certain number of generations, and (3) a preset generation has gone by. Since the philosophy undergoing is the survival of the fittest, the GA tends to yield the global minimum and is suitable for the inverse problem being investigated. For inverse scattering purposes, the regular shape expansion technique is introduced to work with the

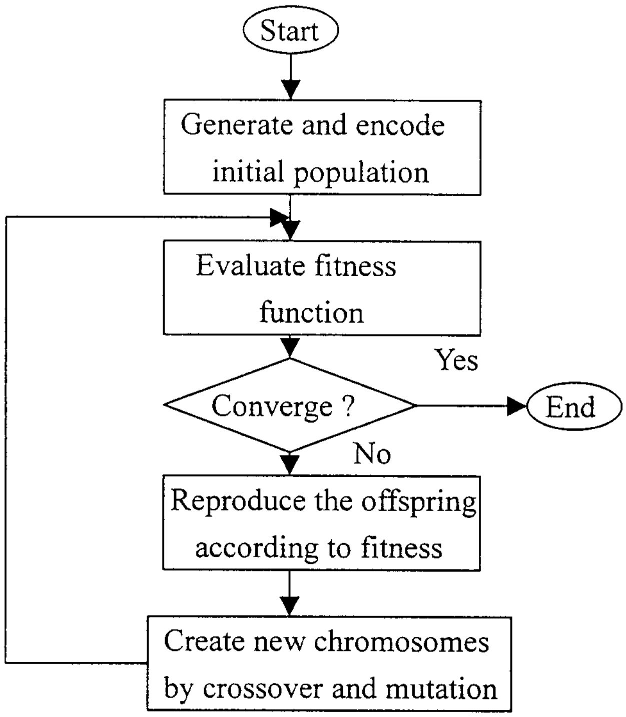

Figure 2. Flow chart of the GA.

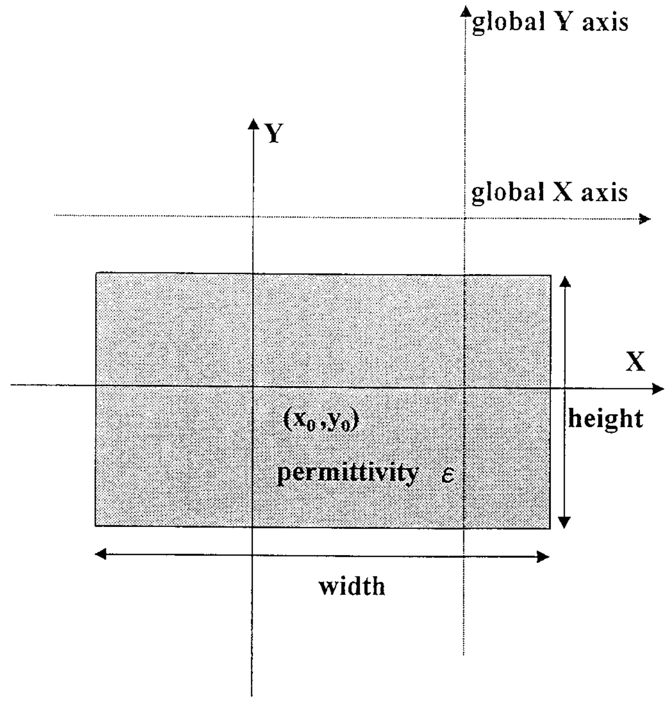

Figure 4. Five parameters are used in the structure of a rectangular dielectric cylinder.

GA. As depicted in Figure 3, the idea of regular shape expansion is to expand an arbitrary multilayer dielectric object in terms of some possibly overlapped regular-shape cylinders (only rectangles are used in this paper). The computation of cell permittivities for the overlapped and nonoverlapped regions is also shown in Figure 3. The cells that are covered by the individual rectangles have permittivity ε_1 and ε_2, respectively, while the cells that reside in the possibly overlapped regions have the average permittivity $(\varepsilon_1 + \varepsilon_2)/2$.

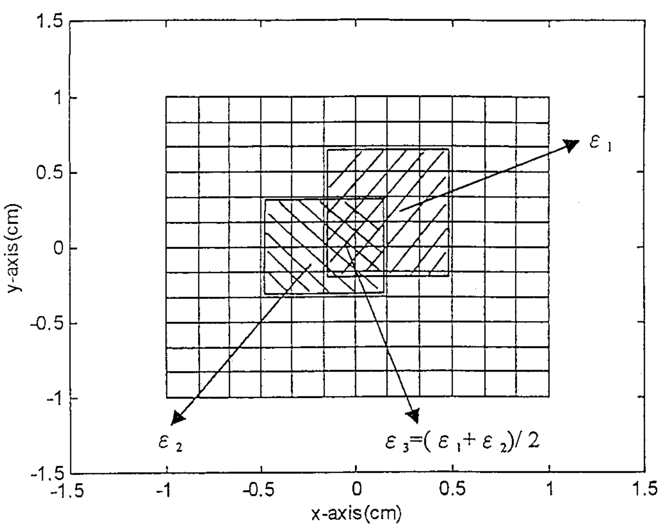

Figure 3. A multilayered dielectric object of arbitrary shape is expanded in terms of some overlapped rectangular-shape cylinders. The average permittivity is used for the overlapped region.

Note that in order to identify each regular/rectangular dielectric cylinder used in shape expansion, there are five parameters to be determined (Fig. 4). They are the center coordinates (x_0, y_0), the width, the height, and the permittivity of the rectangular cylinder. In this method, the parameter number of the GA is independent of the test domain division and is equal to $5 \times M$, where M is the rectangle number used for shape expansion. Since M is directly related to the reconstruction quality and the inversion time, a suitable value for M needs to be carefully set before pursuing the inversion. For the examples studied in this paper, the proposed method can yield the correct permittivity, shape, position, and size of each layer of an unknown layered object beginning with some randomly generated population, without any prior topological information.

III. NUMERICAL RESULTS

The reconstruction of various homogeneous and layered objects illuminated by an electromagnetic TM plane wave is presented. The frequency of the incident wave is 3 GHz, i.e., the wavelength is 0.1 m. The test domain is 0.03×0.03 m^2. The scattered field is measured in the air along a circle of radius 0.3 m around the object. To ensure that the number of independent data measured exceeds the number of independent parameters, the object is illuminated by the incident waves from six different directions. Sixty measurement points of equal spacing along the circle are used for each incident angle. There are totally 360 measured data used in each inversion.

For numerical simulation purposes, the scattered E field is obtained by solving Eq. (6) and Eq. (7) with a 24×24 partition of the test domain, with some noise added to the scattered E field to mimic realistic experimental data. The proposed method is used to solve the inverse problem, for which the GA parameters used are 1,200 for population size, 0.8 for crossover probability, 0.01 for mutation probability, and 15 for coding length. The above probability values

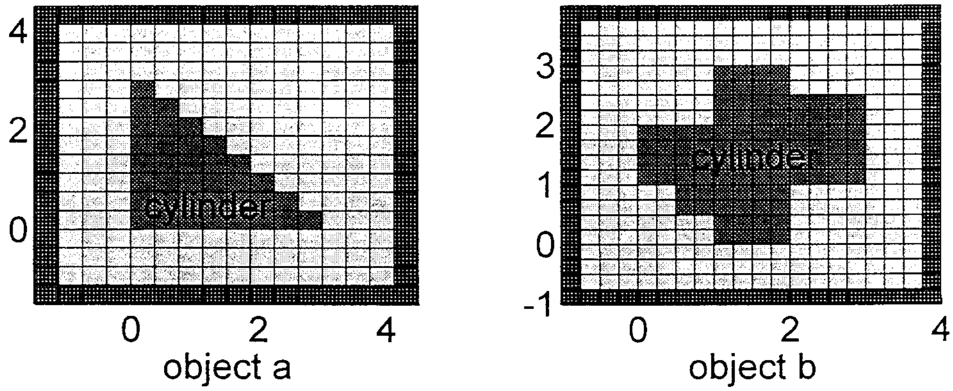

Figure 5. Cross-section shape of homogeneous dielectric objects A and B investigated in this paper.

for crossover and mutation are not critical for this method. In other words, the convergence of the GA does not depend on the choice of these parameters. The fitness function is defined to be inversely proportional to the sum of squared residuals for the measured data as follows:

$$fitness = \left\{ \frac{1}{M_t} \sum_{m=1}^{M_t} \left| E_s^{\exp}(m) - E_s^{sca}(m) \right|^2 \Big/ \left| E_s^{\exp}(m) \right|^2 \right\}^{-0.5}$$

where $M_t = 360$ is the total number of measurements, $E_s^{\exp}(m)$ is the measured scattered field, and $E_s^{sca}(m)$ is the calculated scattered field for each individual.

We now present the reconstructing results of several testing objects for which a very low additive noise level $\sim 10^{-5}$ is set. The testing cases include homogeneous (single layer) and multilayer dielectric objects. Figure 5 shows the shapes of homogeneous dielectric objects A and B, both with relative permittivity 4. Figure 6 shows the reconstructed details for the triangular object A with 1.6% error for relative permittivity at the 100th generation. The shape and permittivity error vs. generation are shown in Figure 7, where the shape error is defined as the ratio of the number of cylinder cells at the right position over the number of cylinder cells of desired shape. The object shape usually converges faster than the permittivity does, which is quite reasonable. Note that eight rectangles are employed to expand the triangular shape of object A, i.e., $M = 8$ is set and the test domain is divided into 8×8 small cells for inversion. Thus, there are 40 parameters such that the GA must search through a 40-D hypersurface to yield the final solution. The convergence of the GA does not depend on any specific initial guess for the dielectric image. For the practical optimization process, if no prior topological information is available, the user may begin at $M = 2$ and gradually increase M to go through the searching process until no further improvement is achieved. As a second example, Figure 8 shows the reconstructed details for the arbitrary-shape object B with 1.08% error for relative permittivity at the 75th generation. The shape and permittivity error vs. generation is shown in Figure 9, where the object shape again converges faster than the permittivity does. In this case, $M = 4$ is set and the test domain is divided into 8×8 small cells.

We now present the testing results of layered objects. The original and reconstructed results for a three-layer pyramid-like object C are shown in Figure 10. In practice, M should be set equal to 3 for

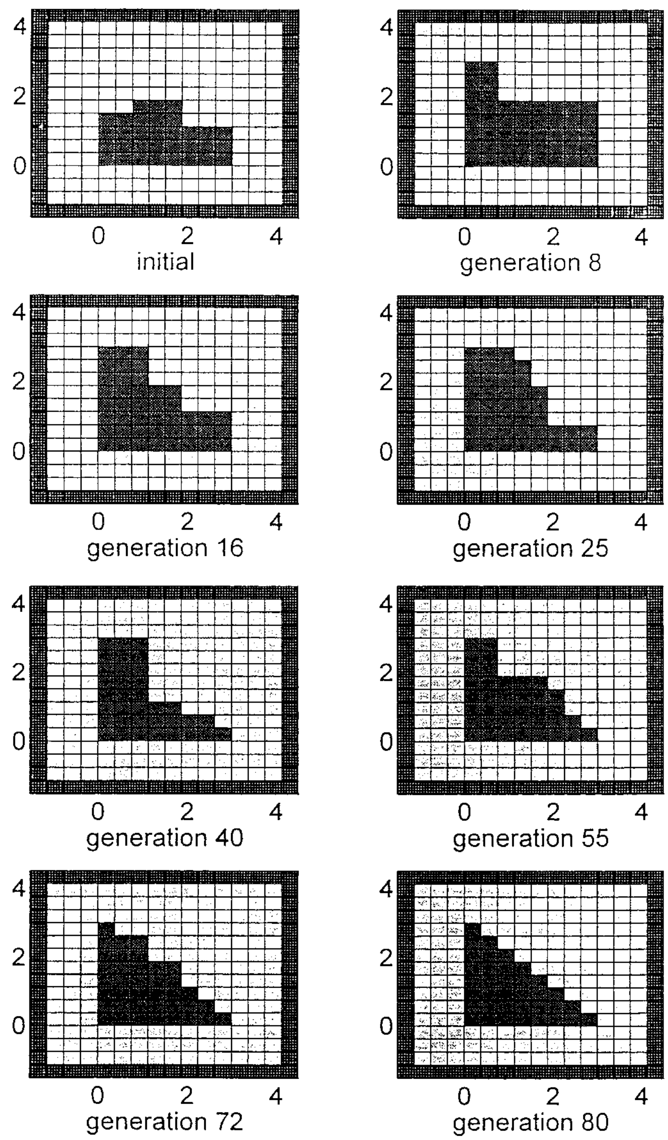

Figure 6. Reconstructed shapes at different generations for object A.

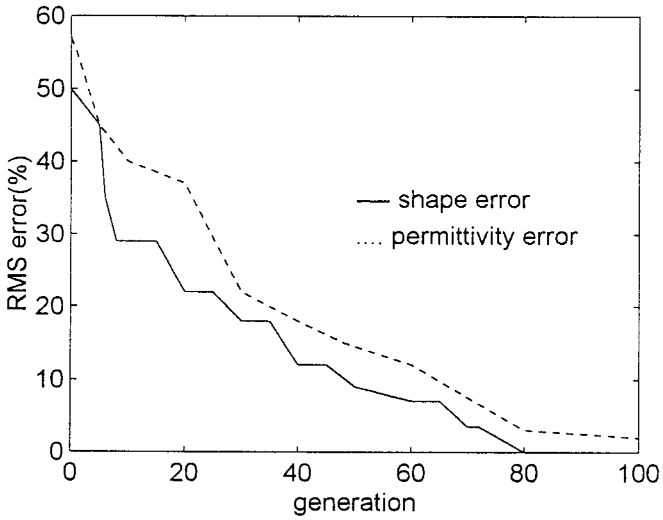

Figure 7. The shape and permittivity error vs. generation for object A.

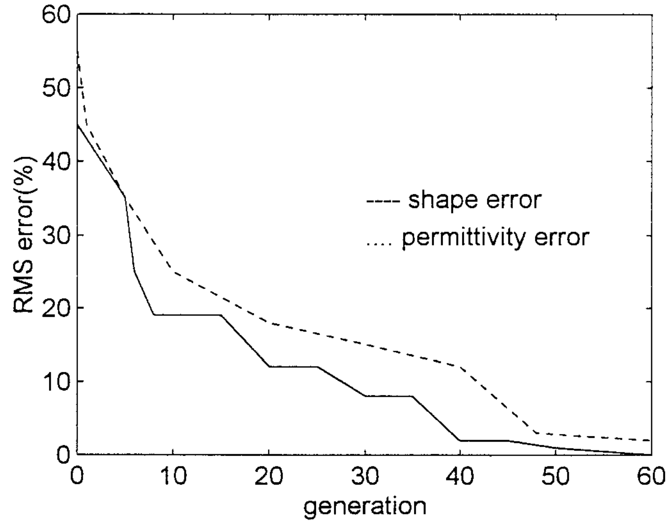

Figure 9. The shape and permittivity error vs. generation for object B.

best inversion. However, for demonstration purposes, $M = 6$ is set. The reconstructed details for object C are shown in Figure 11 with 0.62% error for relative permittivity at the 60th generation. Besides, the test domain is divided into 10×10 small cells in this case. Object D is another layered object studied as shown in Figure 12. It is a five-layer square object for which $M = 5$ is set for inversion. The reconstructed details for object D are shown in Figure 13 with a relative error of 0.69% for relative permittivity at the 50th generation. The convergence of the GA does depend on the complexity of the object. For the layered objects studied, a large population size (usually $\geq$800) is required to achieve convergence, whereas, this can be relaxed for homogeneous objects.

To investigate the effects of noise level on the reconstruction quality by using this method, Gaussian random noise with zero mean is intentionally added to the scattered field and the reconstructed permittivity error against noise level is plotted as shown in Figure

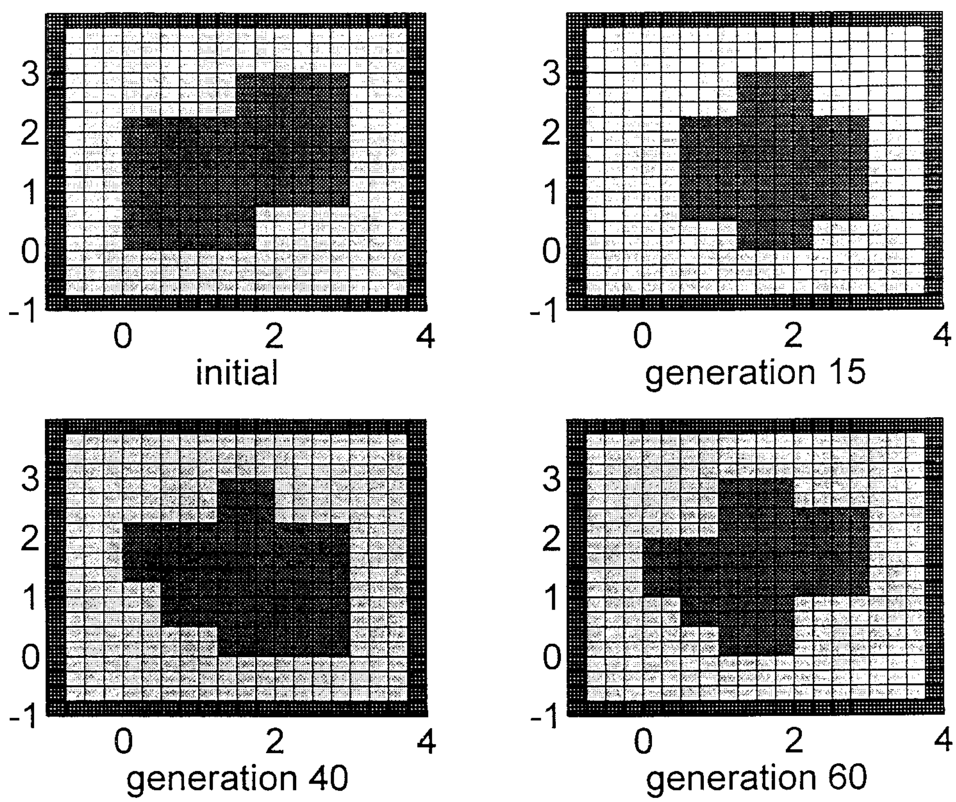

Figure 8. Reconstructed shapes at different generations for object B.

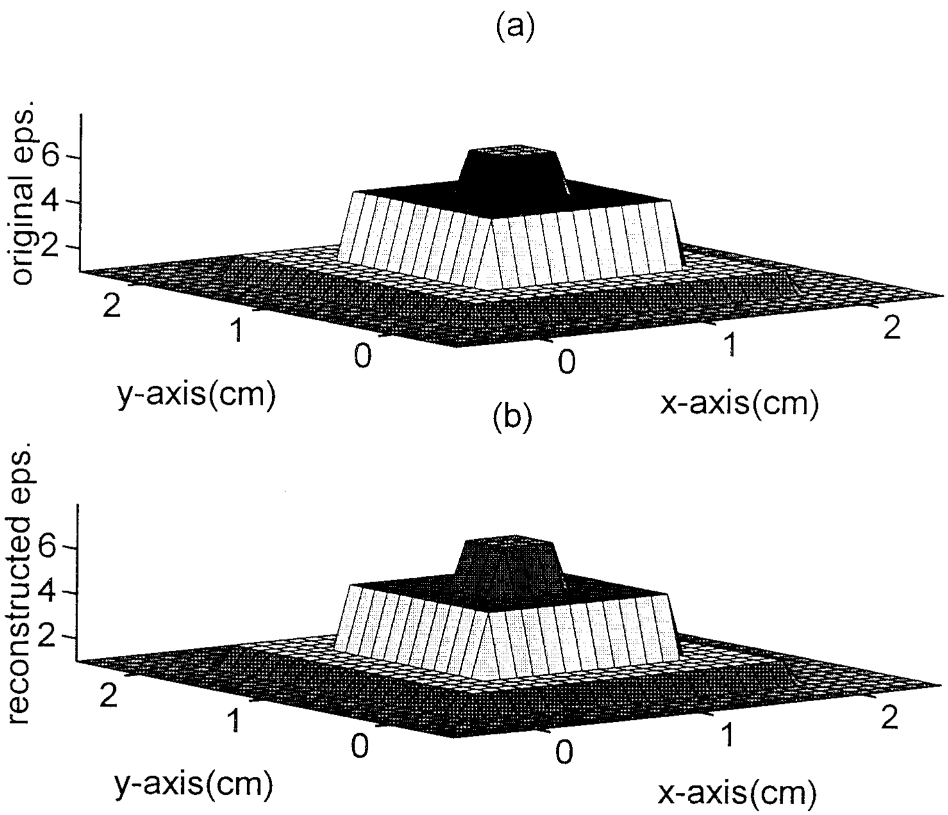

Figure 10. The original and reconstructed results for a three-layer object C.

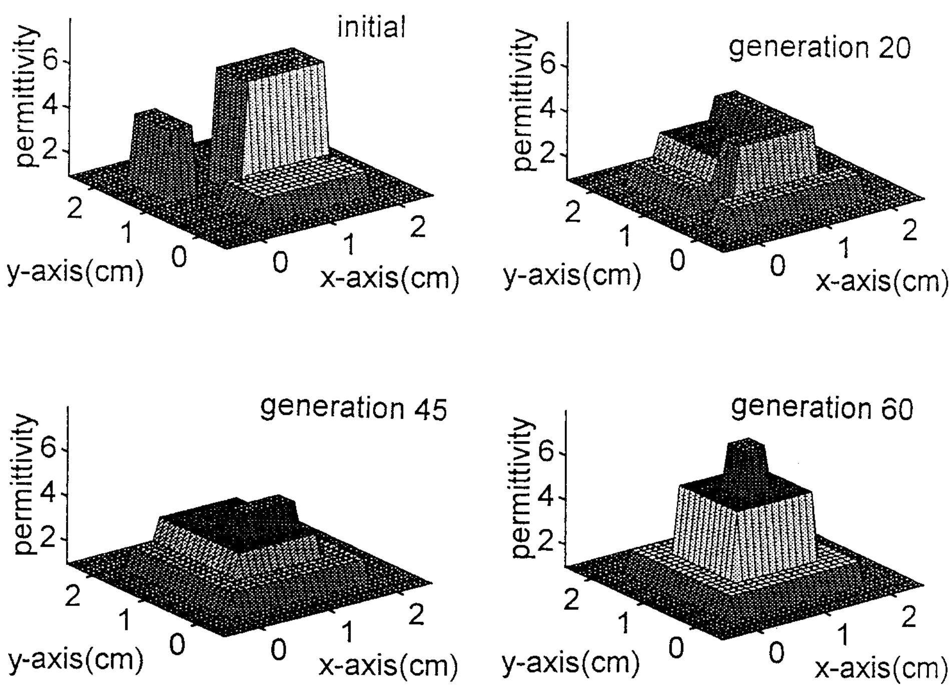

Figure 11. Reconstructed shapes at different generations for object C.

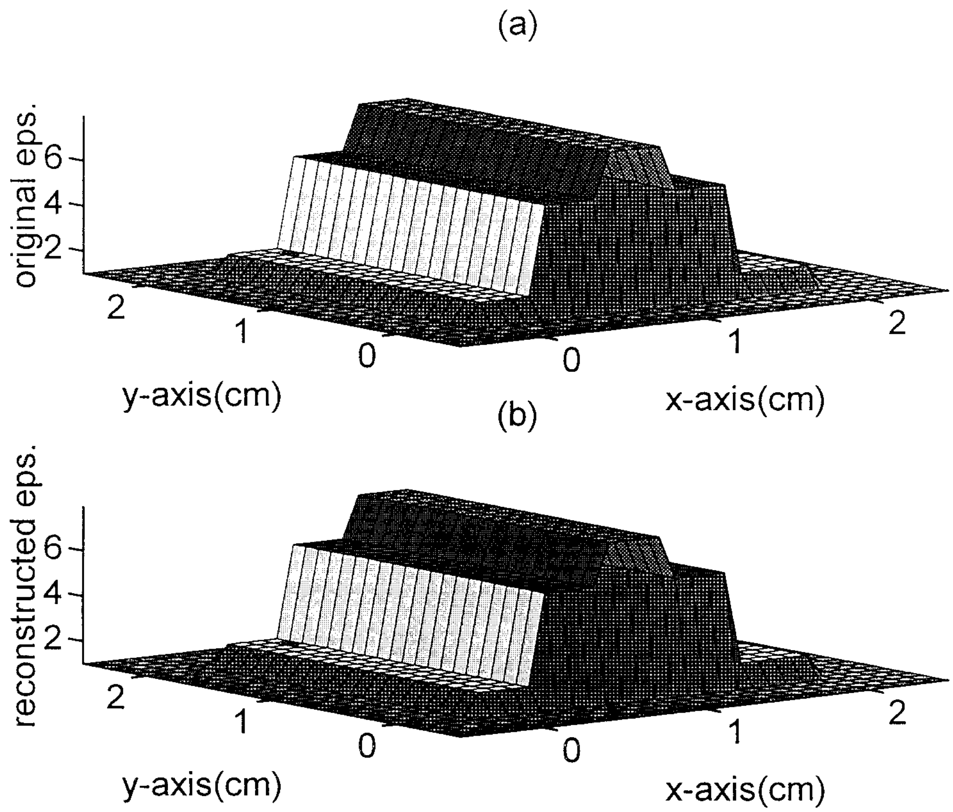

Figure 12. The original and reconstructed results for a five-layered square object D.

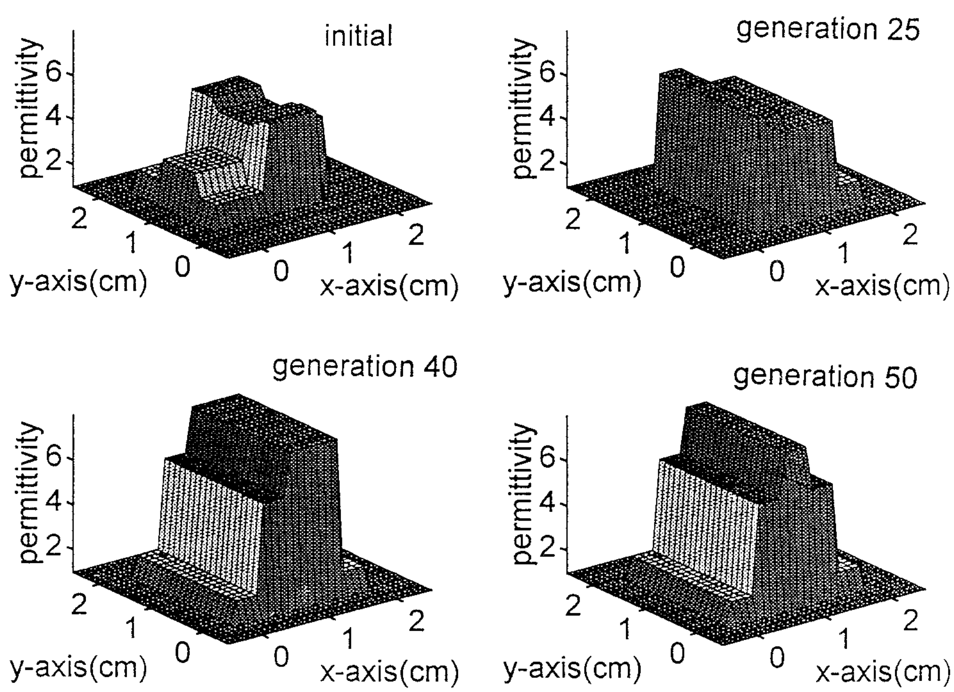

Figure 13. Reconstructed shapes at different generations for object D.

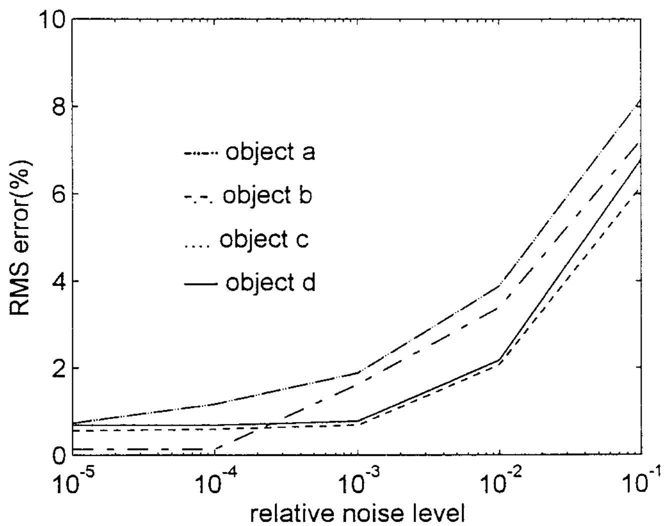

Figure 14. Reconstructed permittivity error against relative noise level for objects A, B, C, and D.

14. It is obvious that the objects can be reconstructed quite well with permittivity error $\leq 7\%$ as long as the relative noise level is ≤ 0.1 (-20 dB), which is a quite satisfactory result. When the noise level is slightly greater than -20 dB, the relative permittivity error would be greater than 10% and become unacceptable, although the convergence of the object shape can still be achieved.

The inversion is a time-consuming process because the GA requires a lot of function calls (1,200 in our case) per generation, each one being computationally expensive for this kind of problem. However, this may be largely improved if the parallel GA is implemented. One advantage for using the GA is that it allows a natural implementation on parallel computers, which seems to be the only possible way of dealing with a lot of data in a reasonable time. Thus, the regular shape expansion technique is well suited for inversion of a layered dielectric object with arbitrary shape.

IV. CONCLUSIONS

We have reported a novel method combining the GA and the regular shape expansion technique to successfully reconstruct the shape, location, size, and permittivity of each layer of a layered dielectric object. This method always converges to the global optimum without any priori topological information, whereas other gradient-type methods usually get trapped in local extremes. Besides, good reconstruction is achieved even when the scattered field is contaminated with random noise. Numerical simulation shows that any multilayer object can be reconstructed quite well as long as the noise level is ≤ -20 dB. Future works may incorporate parallel GA to accelerate the inversion process.

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